

How to understand the lightest scalars*

Nils A. Törnqvist**

Physics Department, POB 9, FIN-00014, University of Helsinki, Finland

Based on previous papers I discuss how I understand the lightest scalars using a general coupled channel model. This model includes all light two-pseudoscalar thresholds, constraints from Adler zeroes, flavour symmetric couplings, unitarity and physically acceptable analyticity. One finds that with a large coupling there can appear two physical resonance poles on the second sheet although only one bare quark-antiquark state is put in. The $f_0(980)$ and $f_0(1370)$ resonance poles are thus in this model two manifestations of the same strange-antistrange quark state. On the other hand, the isoscalar state containing u and d quarks becomes (when unitarized and strongly distorted by hadronic mass shifts) a very broad resonance, with its pole at 470-i250 MeV. This is the sigma meson required by models for spontaneous breaking of chiral symmetry.

§1. Introduction

This talk is based on my previous papers^{1),2),3),4)}, including a few new comments. I try to explain how one can understand the controversial light scalar mesons, and show that one can describe the S-wave data on the light $q\bar{q}$ nonet with a model which includes most well established theoretical constraints:

- Adler zeroes as required by chiral symmetry,
- all light two-pseudoscalar (PP) thresholds with flavor symmetric couplings in a coupled channel framework
- physically acceptable analyticity, and
- unitarity.

A unique feature of this model is that it simultaneously describes the whole scalar nonet and one obtains a good representation of a large set of relevant data. Only six parameters, which all have a clear physical interpretation, are needed: an overall coupling constant ($\gamma = 1.14$), the bare mass of the $u\bar{u}$ or $d\bar{d}$ state, the extra mass for a strange quark ($m_s - m_u = 100$ MeV), a cutoff parameter ($k_0 = 0.56$ GeV/c), an Adler zero parameter for $K\pi$, and a phenomenological parameter enhancing the $\eta\eta'$ couplings.

§2. Understanding the S-waves

In Figs. 1-3 we show the obtained fits to the $K\pi$, $\pi\pi$ S-waves and to the $a_0(980)$ resonance peak in $\pi\eta$. The partial wave amplitude is in the case of one $q\bar{q}$ resonance,

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** email: Nils.Tornqvist@Helsinki.fi

such as the $a_0(980)$ can be written

$$A(s) = -Im\Pi_{\pi\eta}(s)/[m_0^2 + Re\Pi(s) - s + iIm\Pi(s)], \quad (2.1)$$

where

$$Im\Pi(s) = \sum_i Im\Pi_i(s) \quad (2.2)$$

$$= - \sum_i \gamma_i^2 (s - s_{A,i}) \frac{k_i}{\sqrt{s}} e^{-k_i^2/k_0^2} \theta(s - s_{th,i}) ,$$

$$Re\Pi_i(s) = \frac{1}{\pi} \mathcal{P} \int_{s_{th,1}}^{\infty} \frac{Im\Pi_i(s')}{s' - s} ds' . \quad (2.3)$$

Here the coupling constants γ_i are related by flavour symmetry and the OZI rule, such that there is only one over all parameter γ . The $s_{A,i}$ are the positions of the Adler zeroes, which normally are $s_{A,i} = 0$, except $s_{A,\pi\pi} = m_\pi^2/2$, and $s_{A,K\pi}$, which is a free parameter.

In the flavourless channels the situation is a little more complicated than eqs. (1-3) since one has both $u\bar{u} + d\bar{d}$ and $s\bar{s}$ states, requiring a two dimensional mass matrix (See Ref. ¹). Note that the sum runs over all light PP thresholds, which means three for the $a_0(980)$: $\pi\eta$, $K\bar{K}$, $\pi\eta'$ and three for the $K_0^*(1430)$: $K\pi$, $K\eta$, $K\eta'$, while for the f_0 's there are five channels: $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$, $\eta'\eta'$. Five channels means the amplitudes have $2^5 = 32!$ different Riemann sheets, and in principle there can be poles on each of these sheets. In Fig. 4 we show as an example the running mass, $m_0^2 + Re\Pi(s)$, and the width-like function, $Im\Pi(s)$, for the I=1 channel. The crossing point of the running mass with s gives the 90° mass of the $a_0(980)$. The magnitude of the $K\bar{K}$ component in the $a_0(980)$ is determined by $-\frac{d}{ds}Re\Pi(s)$ which is large in the resonance region just below the $K\bar{K}$ threshold. These functions fix the PWA of eq.(1) and Fig. 3. In Fig. 5 the running mass and width-like function for the strange channel are shown. These fix the shape of the $K\pi$ phase shift and absorption parameters in Fig. 1.

Four out of our six parameters are fixed by the $K\pi$ data leaving only $m_s - m_u = 100$ MeV to "predict" the $a_0(980)$ structure Fig. 3, and the parameter β to get the $\pi\pi$ phase shift right above 1 GeV/c. One could discard the β parameter if one also included the next group of important thresholds or pseudoscalar (0^{-+}) -axial (1^{+-}) thresholds, since then the $K\bar{K}_{1B} + c.c.$ thresholds give a very similar contribution to the mass matrix as $\eta\eta'$. As can be seen from Figs. 1-3 the model gives a good description of the relevant data. In Ref. ¹ I did not look for the broad and light sigma on the second sheet in my model, since I looked for only those poles which are nearest to the physical region, and which could complete the light $q\bar{q}$ scalar nonet. I found parameters for these close to the conventional lightest scalars in the PDG tables for the $f_0(980)$, $f_0(1300)$, $a_0(980)$ and $K_0^*(1430)$. Only a little later I realized with Roos²) that I had missed the σ , and that both my $f_0(980)$ and $f_0(1300)$, in fact, originated from the same $s\bar{s}$ input bare state.

Table I. Resonances in the S-wave $PP \rightarrow PP$ amplitudes ¹ The first resonance is the σ which we name here $f_0(\approx 500)$. The two following are both manifestations of the same $s\bar{s}$ state. The $f_0(980)$ and $a_0(980)$ have no approximate Breit Wigner-like description, and the Γ_{BW} given for $a_0(980)$ is rather the peak width. The mixing angle δ_S for the $f_0(\approx 500)$ or σ is with respect to $u\bar{u} + d\bar{d}$, while for the two heavier f_0 's it is with respect to $s\bar{s}$.

resonance	m_{BW}	Γ_{BW}	$\delta_{S,BW}$	Comment
$f_0(\approx 500)$	860	880	$(-9 + i8.5)^\circ$	The σ meson.
$f_0(980)$	-	-	-	First near $s\bar{s}$ state
$f_0(1300)$	1186	360	$(-32 + i1)^\circ$	Second near $s\bar{s}$ state
$K_0^*(1430)$	1349	498	-	The $s\bar{d}$ state
$a_0(980)$	987	≈ 100	-	First I=1 state

Table II. The pole positions of the same resonances as in Table 1. The last entry is an image pole of the $a_0(980)$, which in an improved fit could represent the $a_0(1450)$. The $f_0(1300)$ and $K_0^*(1430)$ poles appear simultaneously on two sheets since the $\eta\eta$ and the $K\eta$ couplings, respectively, nearly vanish. The mixing angle δ_S for the $f_0(\approx 500)$ or σ is with respect to $u\bar{u} + d\bar{d}$, while for the two heavier f_0 's it is with respect to $s\bar{s}$.

resonance	$s_{\text{pole}}^{1/2}$	$[\text{Res}_{\text{pole}}]^{1/2}$	$\frac{-\text{Im } s_{\text{pole}}}{m_{\text{pole}}}$	$\delta_{S,\text{pole}}$	Sheet
$f_0(\approx 500)$	$470 - i250$	397	590	$(-3.4 + i1.5)^\circ$	II
$f_0(980)$	$1006 - i17$	1006	34	$(0.4 + i39)^\circ$	II
$f_0(1300)$	$1214 - i168$	1202	338	$(-36 + i2)^\circ$	III,V
$K_0^*(1430)$	$1450 - i160$	1441	320	-	II,III
$a_0(980)$	$1094 - i145$	1084	270	-	II
$a_0(1450)?$	$1592 - i284$	1566	578	-	III

§3. One $q\bar{q}$ pole can give rise to two resonances

As pointed out by Morgan and Pennington ⁵⁾ for each $q\bar{q}$ state there are, in general, apart from the nearest pole also image poles, usually located far from the physical region. As explained in more detail in Ref. ²⁾ some of these can (for a large enough coupling and sufficiently heavy threshold) come so close to the physical region that they make new resonances. And, in fact, there are more than four physical poles with different isospin, in the output spectrum of my model, although only four bare states are put in! In Table 2 I list the significant pole positions.

All these poles are manifestations of *the same nonet* ²⁾. The $f_0(980)$ and the $f_0(1300)$ turn out to be two manifestations of the same $s\bar{s}$ state. (See Ref ²⁾ for details). There can be two crossings with the running mass $m_0^2 + \text{Re}\Pi(s)$, one near the threshold and another at higher mass, and each one is related to a different pole at the second sheet (or if the coupling is strong enough the lower one could even become a bound state pole, below the threshold, on the first sheet).

Similarly the $a_0(980)$ and the $a_0(1450)$ could be two manifestations of the $u\bar{d}$ state. After I had realized this I, of course, had to find the $u\bar{u} + d\bar{d}$ pole of my model. Then I found my light and broad σ .

§4. The light σ resonance

A light scalar-isoscalar meson (the σ), with a mass of twice the constituent u, d quark mass coupling strongly to $\pi\pi$ is of importance in many models for spontaneous breaking of chiral symmetry, and for the understanding of all hadron masses, since the σ generates constituent quark masses different from chiral quark masses. Thus most of the nucleon mass can be generated by its coupling to the σ , which acts like an effective Higgs-like boson for the hadron spectrum. However, when I worked on this model the lightest well established mesons in the Review of Particle Physics did not include the σ . The lightest isoscalars at that time, which had the quantum numbers of the σ were the $f_0(980)$ and $f_0(1300)$. These do not have the right properties being both too narrow. Furthermore, $f_0(980)$ couples mainly to $K\bar{K}$, and $f_0(1300)$ is too heavy.

Thus I found it exciting that the important pole in my $u\bar{u} + d\bar{d}$ channel turned out to be the first pole in Table 2, which I believe is the long sought for $\sigma = f_0(\approx 600)$ with twice the constituent quark mass as in the famous Nambu relation ($m_\sigma \approx 2m_q$). It gives rise to a very broad Breit-Wigner-like background, dominating $\pi\pi$ amplitudes below 900 MeV. It has the right mass and width and large $\pi\pi$ coupling as predicted by the σ model.

The existence of this meson becomes evident if one studies the $u\bar{u} + d\bar{d}$ channel separately. This can be done within the model, preserving unitarity and analyticity, by letting the s quark (and K, η etc.) mass go to infinity. Thereby one eliminates the influence from $s\bar{s}$ and $K\bar{K}$ channels, which perturb $\pi\pi$ scattering very little through mixing below 900 MeV. The $u\bar{u} + d\bar{d}$ channel is then seen to be dominated by the sigma below 900 MeV. For more details see Ref. ⁴⁾.

Isgur and Speth ⁶⁾ criticised this result claiming that details of crossed channel exchanges, in particular ρ exchange, are important. In our reply ⁶⁾ to this we emphasized the well known result from dual models, that a sum of s -channel resonances also describes t -channel phenomena. In my model the crossed channel singularities are represented, although in a very crude way, through the form factor $F(s)$, which in an N/D language is related to the N function. See also Igi and Hikasa ⁷⁾. Improvements to the model can be done by allowing for a more complicated analytic form for $F(s)$.

I believe more important than the details of crossed channel singularities is the fact that I included the chiral (Adler) zeroes. These were absent in my first attempt in 1982 to fit the scalars using a unitarized quark model ¹⁾.

§5. The large mass difference between the $K_0^*(1430)$ and the $a_0(980)$

Many authors argue that the $a_0(980)$ and $f_0(980)$ are not $q\bar{q}$ states, since in addition to being very close to the $K\bar{K}$ threshold, they are much lighter than the first strange scalar, the $K_0^*(1430)$. Naively one expects a mass difference between the strange and nonstrange meson to be of the order of the strange-nonstrange quark mass difference, or a little over 100 MeV. This is also one of the reasons why some authors want to have a lighter strange meson, the κ , near 800 MeV. Cherry and

Pennington⁸⁾ recently have strongly argued against its existence.

Figs. 4 and 5 explain why one can easily understand this large mass splitting as a secondary effect of the large pseudoscalar mass splittings, and because of the large mass shifts coming from the loop diagrams involving the PP thresholds. If one puts Figs. 4 and 5 on top of each other one sees that the 3 thresholds $\pi\eta$, $K\bar{K}$, $\pi\eta$ all lie relatively close to the $a_0(980)$, and all 3 contribute to a large mass shift. On the other hand, for the $K_0^*(1430)$ the $SU3_f$ related thresholds ($K\pi$, $K\eta'$) lie far apart from the K_0^* , while the $K\eta$ nearly decouples because of the physical value of the pseudoscalar mixing angle.

§6. Concluding remarks

An often raised question is: Why are the mass shifts required by unitarity so much more important for the scalars than, say, for the vector mesons? The answer is very simple, and there are two main reasons:

- The scalar coupling to two pseudoscalars is very much larger than the corresponding coupling for the vectors, both experimentally and theoretically (e.g. spin counting gives 3 for the ratio of the two squared couplings).
- For the scalars the thresholds are S-waves, giving nonlinear square root cusps in the $\Pi(s)$ function, whereas for the vectors the thresholds are P-waves, giving a smooth k^3 angular momentum and phase space factor.

One could argue that the two states $f_0(980)$ and $a_0(980)$ are a kind of $K\bar{K}$ bound states (c.f. Ref.⁹⁾), since these have a large component of $K\bar{K}$ in their wave functions. However, the dynamics of these states is quite different from that of normal two-hadron bound states. If one wants to consider them as $K\bar{K}$ bound states, it is the $K\bar{K} \rightarrow s\bar{s} \rightarrow K\bar{K}$ interaction which creates their binding energy, not the hyperfine interaction as in Ref.⁹⁾. Thus, although they may spend most of their time as $K\bar{K}$, they owe their existence to the $s\bar{s}$ state. Therefore, it is more natural to consider the $f_0(980)$ and $f_0(1300)$ as two manifestations of the same $s\bar{s}$ state.

The wave function of the $a_0(980)$ (and $f_0(980)$) can be pictured as a relatively small core of $q\bar{q}$ of typical $q\bar{q}$ meson size (0.6fm), which is surrounded by a much larger standing S-wave of $K\bar{K}$. This picture also gives a physical explanation of the narrow width: In order to decay to $\pi\eta$ the $K\bar{K}$ component must first virtually annihilate near the origin to $q\bar{q}$. Then the $q\bar{q}$ can decay to $\pi\eta$ as an OZI allowed decay.

References

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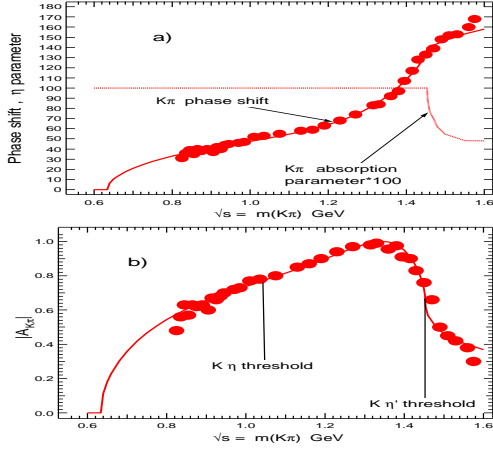


Fig. 1. (a) The $K\pi$ S-wave phase shift and (b) the magnitude of the $K\pi$ partial wave amplitude compared with the model predictions, which fix 4 (γ , $m_0 + m_s$, k_0 and $s_{A,K\pi}$) of the 6 parameters.

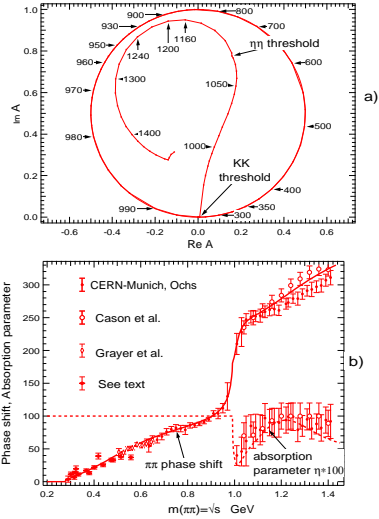


Fig. 2. (a) The $\pi\pi$ Argand diagram and (b) phase shift predictions are compared with data. Note that most of the parameters were fixed by the data in Fig.1. For more details see Ref. ^{1,2}.

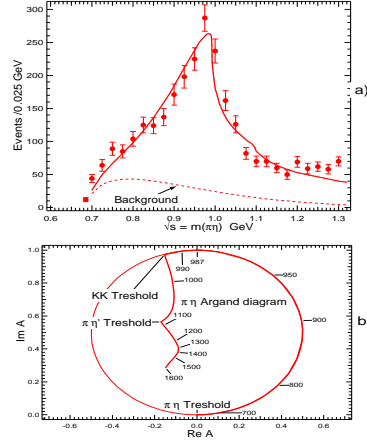


Fig. 3. (a) The $a_0(980)$ peak compared with model prediction and (b) the predicted $\pi\eta$ Argand diagram

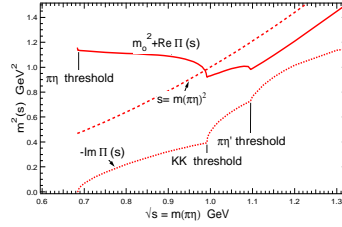


Fig. 4. The running mass $m_0 + \text{Re} \Pi(s)$ and $\text{Im} \Pi(s)$ of the $a_0(980)$. The strongly dropping running mass at the $a_0(980)$ position, below the $K\bar{K}$ threshold contributes to the narrow shape of the peak in Fig. 3a.

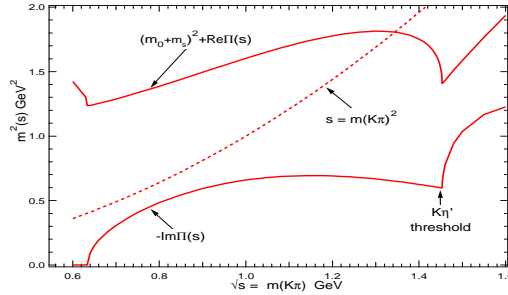


Fig. 5. The running mass and width-like function $\text{Im} \Pi(s)$ for the $K_0^*(1430)$. The crossing of s with the running mass gives the 90° phase shift mass, which roughly corresponds to a naive Breit-Wigner mass, where the running mass is put constant.